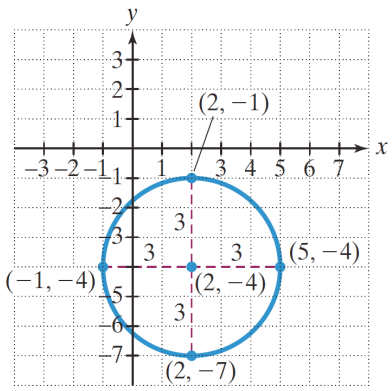
Intermediate Algebra
Final Exam Review

Part One: Do not use a calculator on this portion of the review. Give all answers in exact form, and show all supporting work.

1. *(a)* Write the standard form of the equation of a circle with center (–2, 3) and radius of 4. Graph the circle. *(b)* Make use of completing the square to write the standard form of the equation of a the following circle:
$x^2 + y^2 - 6x + 4y - 23 = 0.$

2. Find the standard form of the equation of the circle graphed to the right:



Problems #3-6: Evaluate without using a calculator.

3. $7\sqrt{64}$
4. $\sqrt[3]{-27}$
5. $\log_2\left(\frac{1}{16}\right)$
6. $e^{\ln 32}$

Problems #7-11: Find the domains of the given functions.

7. $f(x) = \sqrt{5-3x}$
8. $f(x) = 3^{x+1}$
9. $f(x) = \log_2(x-4)$
10. $f(x) = 2x - 7$
11. $f(x) = x^2 + 2$

12. Find the excluded values for the given expression:

$$\frac{x^2 - 25}{x^2 - 2x - 3}$$

Problems #13-30: Solve the given equations. Show all supporting work and give all answers in exact form.

13. $2+\frac{9}{x^2}=\frac{9}{x}$
14. $|4x-2|+5=13$
15. $6\ln(2x)=30$
16. $\sqrt{4-x}=x-2$
17. $\sqrt{4x+1}=6$
18. $\sqrt[3]{x+1}=2$
19. $\sqrt[3]{6x-3}-3=0$
20. $\sqrt{2x-1}-4=-\sqrt{x-4}$
21. $(x+1)^2-12=0$
22. $(3x-2)^2+4=0$
23. $2x^2-4x=3$
24. $5x^2-3=14x$
25. $x^{\frac{2}{3}}-5x^{\frac{1}{3}}=-6$
26. $x^4-3x^2=4$
27. $5^{2x-3}=25$
28. $25^{2x-3}=5$
29. $\log_2(x+1)=4$
30. $\log_2 x+\log_2(x+2)=3$

Problems #31-32: Solve the given inequalities. Show all supporting work.

31. $2x^2-5x-7<0$
32. $\frac{x-3}{x+7}\geq 0$

Problems #33-34: Simplify each of the following. Use absolute values where appropriate.

33. $\sqrt{(3x-8)^2}$

34. $\sqrt[5]{(3x-8)^5}$

Problems #35-36: Convert each radical to a rational exponent and simplify. Write the final answer in radical form. Assume all variables represent positive numbers.

35. $\sqrt[7]{x^2} \bullet \sqrt[6]{x}$

36. $\dfrac{\sqrt[5]{x}}{\sqrt[3]{x^2}}$

Problems #37-38: Simplify the given radicals. Assume all variables represent positive numbers.

37. $\sqrt{50x^3y^4}$

38. $\sqrt[3]{16x^4y^5}$

Problems #39-42: Perform the indicated operations and simplify. Assume all variables represent positive numbers.

39. $\sqrt{12xy} \bullet \sqrt{3y}$

40. $\sqrt[5]{8x^3y^4} \bullet \sqrt[5]{4x^3y^3}$

41. $\sqrt{3}\left(2x+\sqrt{6}\right)$

42. $\left(5-\sqrt{3}\right)\left(6+\sqrt{2}\right)$

Problems #43-45: Rationalize each denominator. Assume all variables represent positive numbers.

43. $\dfrac{5\sqrt{3x}}{\sqrt{y}}$

44. $\dfrac{2+\sqrt{x}}{3-\sqrt{x}}$

45. $\dfrac{4x}{\sqrt[5]{2x^2y^4}}$

Problems #46-48: Express each in terms of i and simplify.

46. $\sqrt{-36}$

47. $3\sqrt{-25}$

48. $\sqrt{-9} \bullet \sqrt{-16}$

Problems #49-52: Perform the indicated operations and simplify. Assume all variables represent positive numbers.

49. $(3+i)^2 + (4-2i)$

50. $\dfrac{12+i}{2-3i}$

51. i^{29}

52. i^{40}

Problems #53-55: Write a quadratic equation with integer coefficients that has the given solutions.

53. $\left\{\dfrac{1}{2}, 3\right\}$

54. $\{-4, 2\}$

55. $\{2i, -2i\}$

Problems #56-58: Calculate the discriminant. Determine the number and the kind of solutions to the quadratic equation.

56. $-2x^2 + 9x + 5 = 0$

57. $5x^2 - 4x = -6$

58. $-9x^2 = 6x - 1$

Problems #59-63: Given $f(x) = 4x^2 - x - 7$, find each of the following.

59. f(4)

60. f(−2)

61. $f\left(\sqrt{2}\right)$

62. $f(2i)$

63. $f(t+i)$

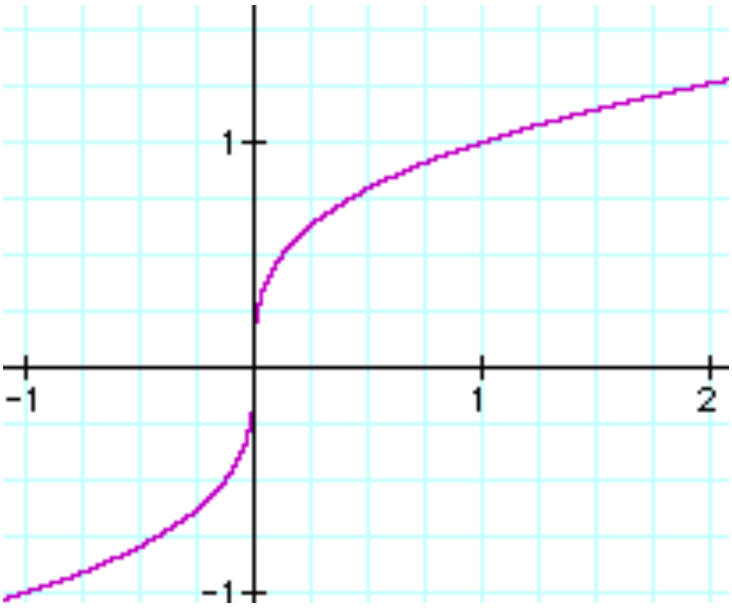
Problems #64-69: Given $f(x) = x^2 - 1$ and $g(x) = 2x - 3$, find each of the following.

64. (f + g)(0)
65. (fg)(x)
66. f(3) + g(–1)
67. $(f \circ g)(2)$
68. $(f \circ g)(x)$

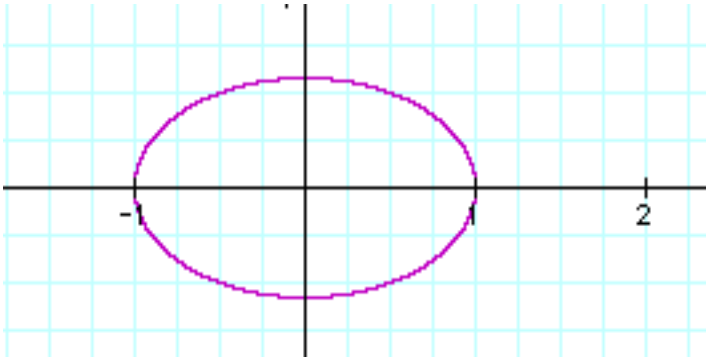
69. State the Vertical Line Test. What is the Vertical Line Test used for?
70. State the Horizontal Line Test. What is the Horizontal Line Test used for?

Problems #71-73: For each graph, apply the appropriate test and determine if the graph represents a function and, if it does represent a function, whether the function has an inverse function.

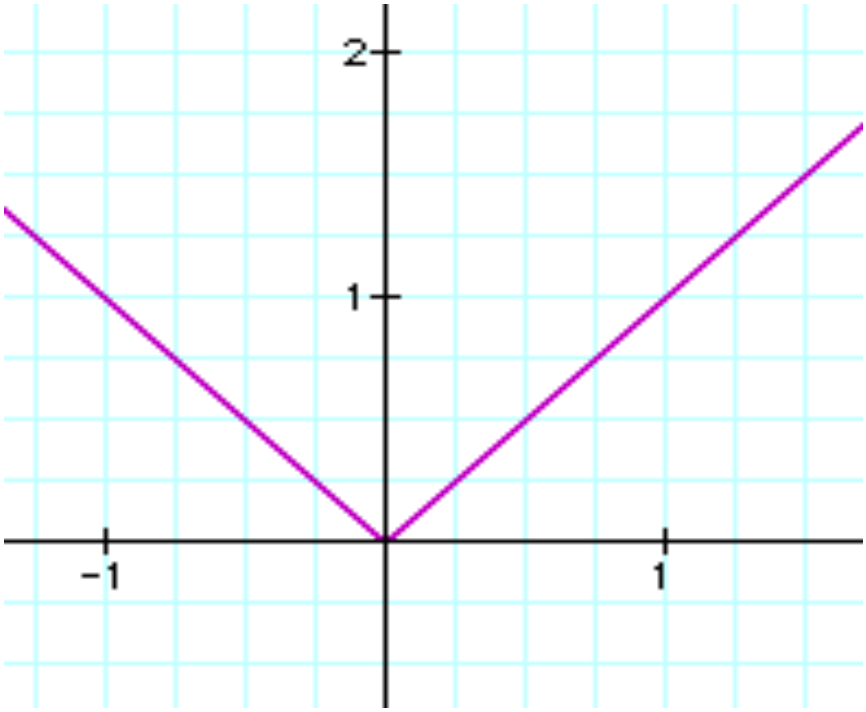
71.



72.

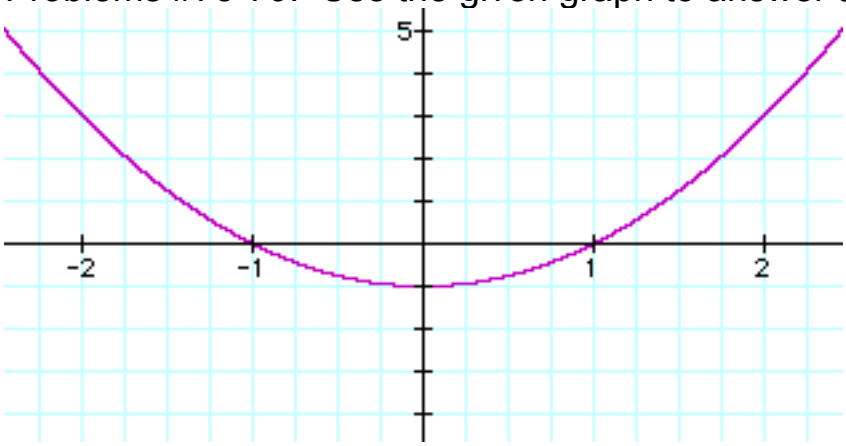


73.



74. Given $f(x)=\frac{5x-7}{6}$, find the inverse function $f^{-1}(x)$.

Problems #75-79: Use the given graph to answer the questions.



75. What is the smallest y-value on the graph (the minimum value for the function)?
76. What is the x-coordinate where the smallest y-value occurs?
77. What are the x-intercepts? (Give your answer in ordered pair form).
78. If x = 0, what is y?
79. If f(x) = 3, what is x?

Problems #80-89: Graph the given functions. Set up a table of coordinates. Find x-intercepts and y-intercepts, and any other important features of the graph. For a parabola, find the vertex. For an exponential function, give the equation of the horizontal asymptote.

80. $f(x) = 2(x-1)^2 + 3$
81. $f(x) = -2(x+2)^2 - 1$
82. $f(x) = x^2 - 6x + 5$
83. $f(x) = -x^2 + 8x - 17$
84. $f(x) = 2^x$
85. $f(x) = 2^{x+3}$
86. $f(x) = 2^x - 1$
87. $f(x) = \log_2 x$
88. $f(x) = \log_{\frac{1}{3}} x$89. $f(x) = \sqrt{x+3}$

Part Two: You may use your calculator on this portion of the review. Show enough work so that I can see where you got your answer. Give exact answers and then use your calculator to approximate to the nearest thousandth (three decimal places).

Problems #90-96: Use your calculator to evaluate each of the following to the nearest thousandth.

90. $\sqrt{9+19}+\sqrt{25}$

91. $(15)^{\frac{4}{7}}$

92. $\sqrt[5]{41}+\sqrt[4]{4}$

93. $e^{1.34}$

94. $\ln 8.3$

95. $\log 16.7$

96. $\log_2 5.78$

Problems #97-101: Solve the given applied problems. Round answers to the nearest tenth.

97. Find the solutions to the given equation:

$$0 = -0.2x^2 + 0.4x + 1$$

98. Find the maximum value of the function and the value of x at which the maximum value occurs:

$$f(x) = -0.2x^2 + 0.4x + 1$$

99. Standing on a platform 50 feet high, a person accidentally fires a gun straight into the air. The function

$$f(t) = -16t^2 + 60t + 50$$

models the bullet's height above the ground, f(t), in feet, t seconds after the gun was fired. How long will it take for the bullet to hit the ground? Use a calculator and round your answer to the nearest tenth of a second.

100. The function $f(t) = 10.1e^{0.005t}$ models the population, f(t), of Los Angeles, California, in millions, t years after 1992. If the growth rate continues into the future, when will the population reach 13 million?

101. The function $f(x) = 2.9\sqrt{x} + 20.1$ models the average height, in inches, of boys who are x months of age, $0 < x < 60$. Find the age at which the average height of boys is 40.4 inches.

102. The function $P(x) = 95 - 30\log_2 x$ models the percentage, P(x), of students who could recall the important features of a lecture as a function of time, where x represents the number of days that have elapsed since the lecture was given. After how many days have the 50% of the students forgotten the important features of the lecture. (Hint: If 50% have forgotten, then 50% could recall.) Round your result to the nearest tenth.

$$\begin{cases} x^2 + y^2 = 28 \\ 2y - x^2 = -4 \end{cases}$$

103. How much will an investment of $10,000 be worth in 5 years if the annual interest rate is 8% and compounding is

a. quarterly?

b. continuously?

Solve each of the following equations:

104. $4^x = 7$

105. $4^{2x-1} = 7$

Use elimination, or substitution to solve each of the following systems:

106. $\begin{cases} x^2 = 2y + 10 \\ 3x - y = 9 \end{cases}$

107. $\begin{cases} (x-2)^2 + (y+3)^2 = 4 \\ x - y = 3 \end{cases}$

108. $\begin{cases} x^2 + y^2 = 4 \\ y^2 - x = 4 \end{cases}$

109. $\begin{cases} y = -x^2 - 2x + 14 \\ y = x^2 - 4x - 10 \end{cases}$

Answers:

1. a. $(x+2)^2+(y-3)^2=16$. b. $(x-3)^2+(y+2)^2=36$.	21. $\left\{-1+2\sqrt{3},-1-2\sqrt{3}\right\}$
2. $(x-2)^2+(y+4)^2=9$.	22. $\left\{\frac{2+2i}{3},\frac{2-2i}{3}\right\}$
3. 56	23. $\left\{\frac{2-\sqrt{10}}{2},\frac{2+\sqrt{10}}{2}\right\}$
4. –3	24. $\left\{-\frac{1}{5},\ 3\right\}$
5. –4	25. $\{27,8\}$
6. 32	26. $\{2,-2,i,-i\}$
7. $\left\{x\middle\vert x\le\frac{5}{3}\right\}=\left(-\infty,\frac{5}{3}\right]$	27. $\left\{\frac{5}{2}\right\}$
8. $\{x\vert\text{all real numbers}\}=(-\infty,\infty)$	28. $\left\{\frac{7}{4}\right\}$
9. $\{x\vert x>4\}=(4,\infty)$	29. $\{15\}$
10. $\{x\vert\text{all real numbers}\}=(-\infty,\infty)$	30. $\{2\}$ (–4 doesn't check)
11. $\{x\vert\text{all real numbers}\}=(-\infty,\infty)$	31. $\left(-1,\frac{7}{2}\right)$
12. $x\neq 3$, $x\neq -1$	32. $(-\infty,-7)\cup[3,\infty)$
13. $\left\{\frac{3}{2},\ 3\right\}$	33. $\lvert 3x-8\rvert$
14. $\left\{\frac{5}{2},\ -\frac{3}{2}\right\}$	34. $3x-8$
15. $\left\{\frac{e^5}{2}\right\}$	35. $\sqrt[42]{x^{19}}$
16. $\{3\}$	36. $\frac{1}{\sqrt[15]{x^7}}$
17. $\left\{\frac{35}{4}\right\}$	37. $5xy^2\sqrt{2x}$
18. $\{7\}$	38. $2xy\sqrt[3]{2xy^2}$
19. $\{5\}$	39. $6y\sqrt{x}$

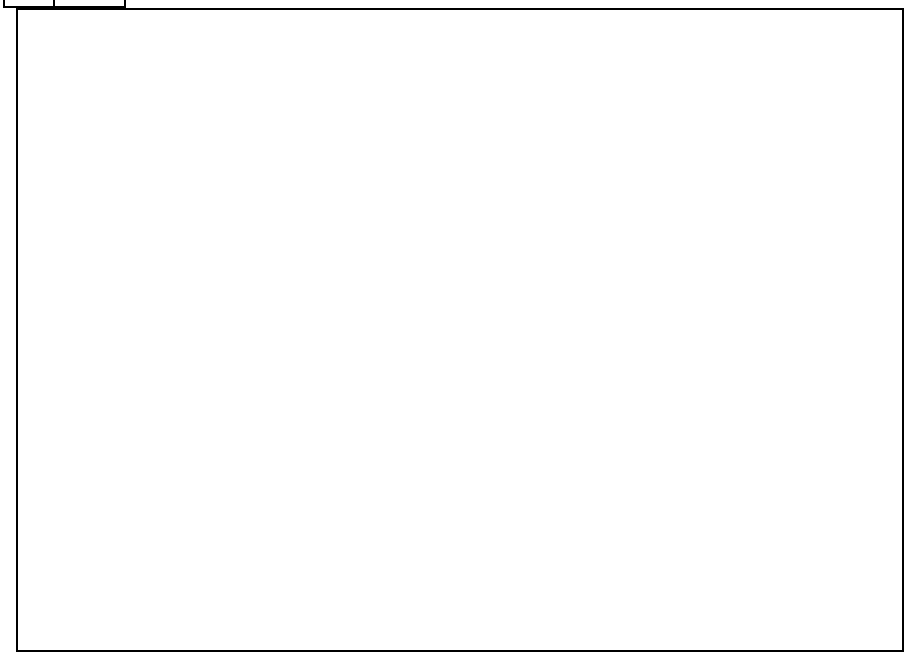
20. $\{5\}$ (85 doesn't check)	40. $2xy\sqrt[5]{xy^2}$
41. $2x\sqrt{3}+3\sqrt{2}$	61. $f(\sqrt{2})=1-\sqrt{2}$
42. $30-6\sqrt{3}+5\sqrt{2}-\sqrt{6}$	62. f(2i) = –23–2i
43. $\frac{5\sqrt{3xy}}{y}$	63. $f(t+i) = 4t^2+8it-t-i-11$
44. $\frac{6+5\sqrt{x}+x}{9-x}$	64. (f+g)(0) = –4
45. $\frac{4x\sqrt[5]{16x^3y}}{2xy}=\frac{2\sqrt[5]{16x^3y}}{y}$	65. $(fg)(x) = 2x^3-3x^2-2x+3$
46. 6i	66. f(3) + g(-1) = 3
47. 15i	67. $(f\circ g)(2)=0$
48. –12	68. $(f\circ g)(x)=4x^2-12x+8$
49. 12 + 4i	69. See pg. 562 in text.
50. $\frac{21+38i}{13}$	70. See pg. 591 in text.
51. i	71. Passes VLT- is a function Passes HLT-has an inverse
52. 1	72. Fails VLT-is not a function and can't have an inverse function.
53. $2x^2-7x+3=0$	73. Passes VLT- is a function Fails HLT-does not have an inverse function
54. $x^2+2x-8=0$ 55. $x^2+4=0$	74. $f^{-1}(x)=\frac{6x+7}{5}$
56. $b^2-4ac=121>0$ two real solutions	75. y = –1
57. $b^2-4ac=-104<0$ two complex sol'ns that are not real	76. x = 0
58. $b^2-4ac=72>0$ two real solutions	77. x = –1, x = 1
59. f(4) = 53	78. y = –1
60. f(–2) = 11	79. x = –2, x = 2

80-89. See graphs on following pages.	97. $\{-1.5,3.5\}$

90. 10.292	98. x = 1.0, max value is y = 1.2
91. 4.7	99. 4.5 sec for the bullet to hit the ground.
92. 3.516	100. 2042 AD is when the pop. will reach 13 million.
93. 3.819	101. age 49 months is when the avg. ht. is 40.4 inches.
94. 2.116	102. 2.8 days
95. 1.223	103. a. $14,895.47; b. $14,918.25
96. 2.531	104. $\{1.404\}$ 105. $\{1.202\}$
106. $\{(4,3),\ (2,-3)\}$	107. $\{(0,3),\ (2,-1)\}$
108. $\{(0,2),\ (0,-2),(-1,\sqrt{3}),\ (-1,-\sqrt{3})\}$	109. $\{(-3,11),\ (4,-10)\}$

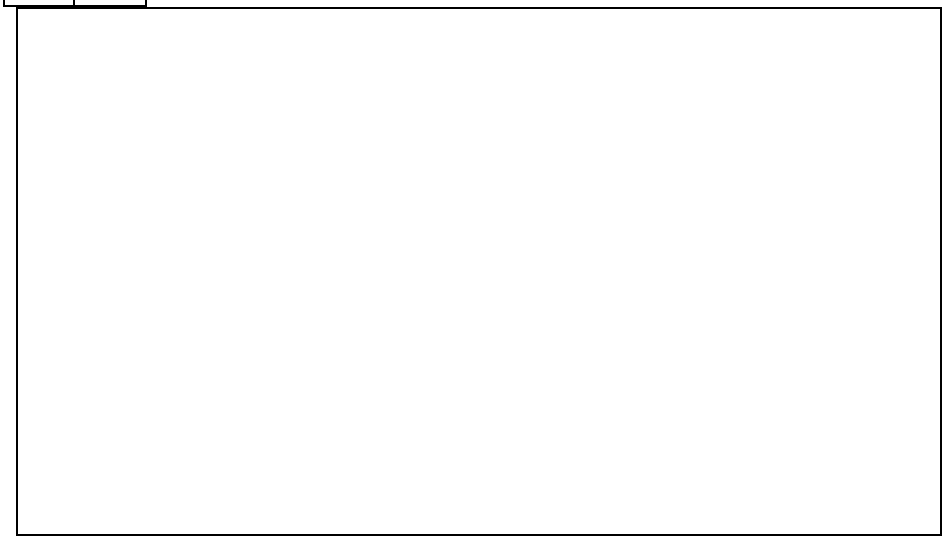
80. V(1,3), a>o so parabola opens up

x	y
0	5
1	3
2	5



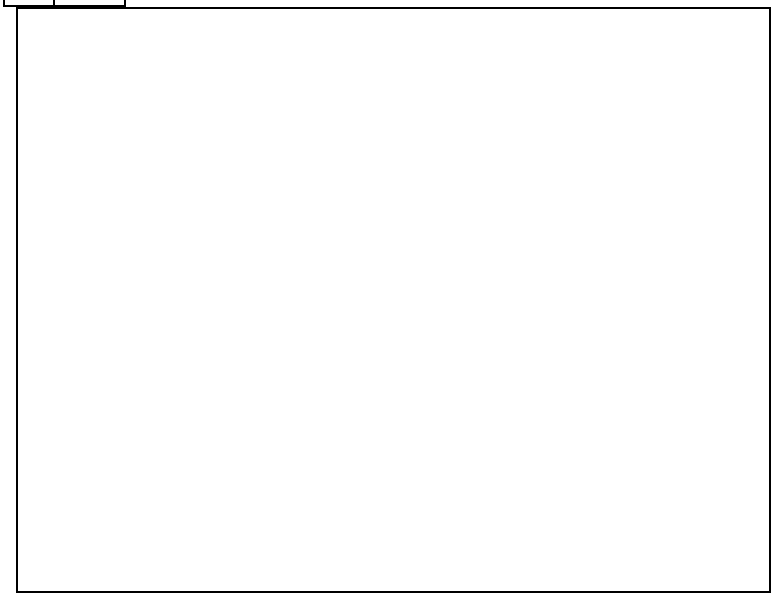
81. V(–2,–1), a<0 so parabola opens downward

x	y
–3	–3
–2	–1
–1	–3



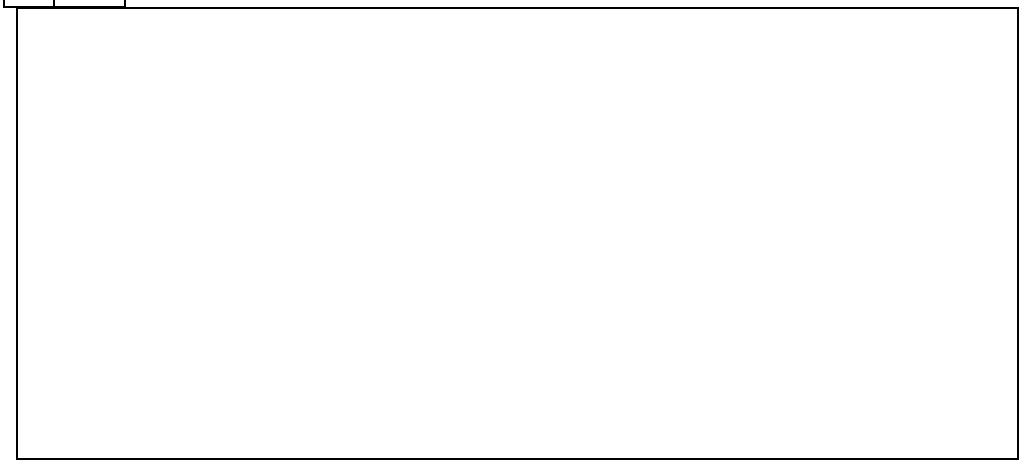
82. V(3,–4), a>0 so the parabola opens upward

x	y
2	–3
3	–4
4	–3



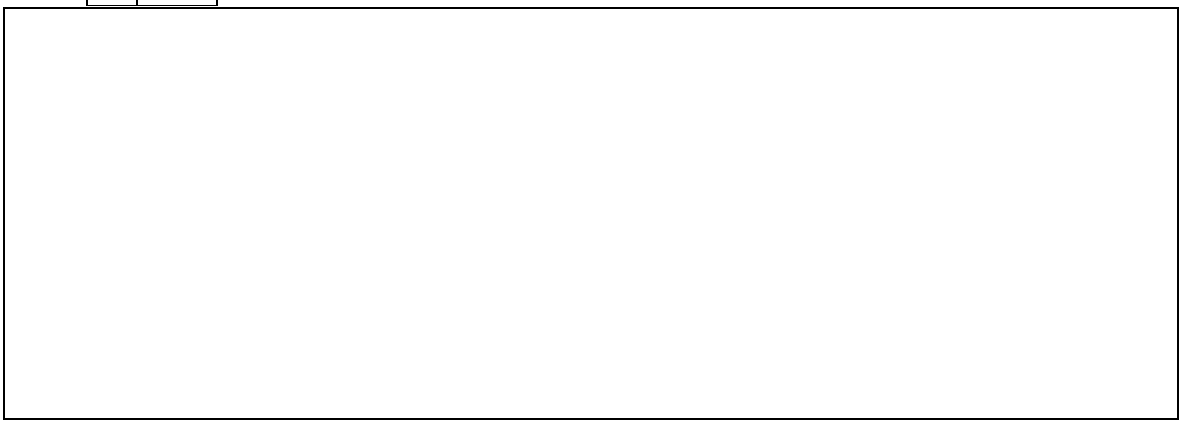
83. V(4,–1), a<0, so the parabola opens downward

x	y
3	–2
4	–1
5	–2



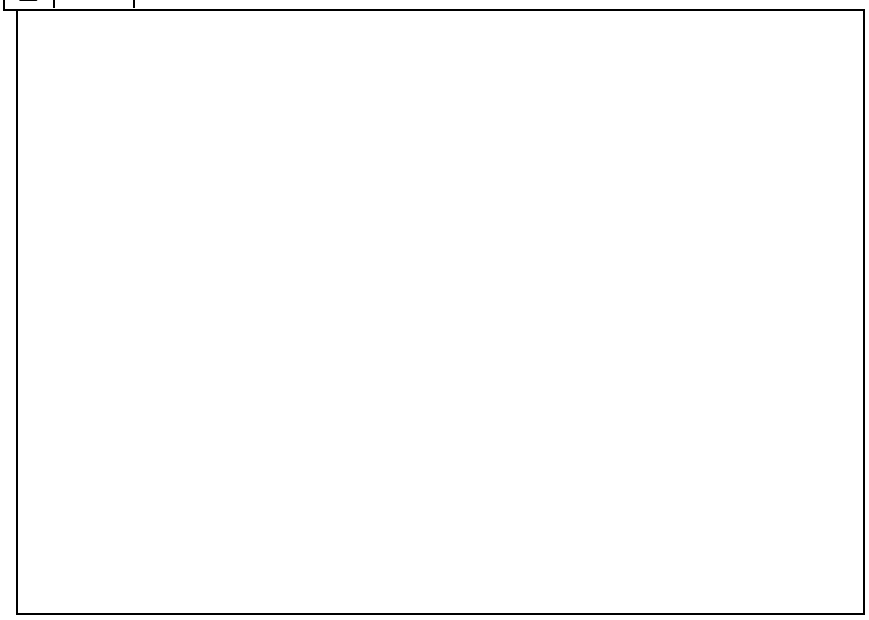
84. exponential graph, H.A. y = 0

x	y
-1	0.5
0	1
1	2



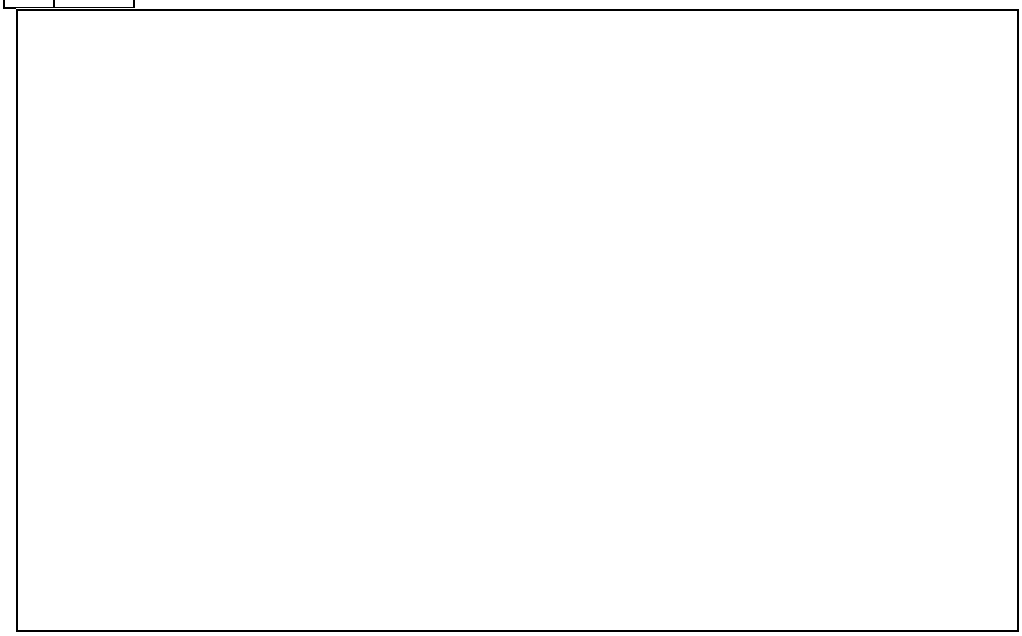
85. exponential graph, shift ☐ three units to the left, H.A. y = 0

x	y
-4	0.5
-3	1
-2	2



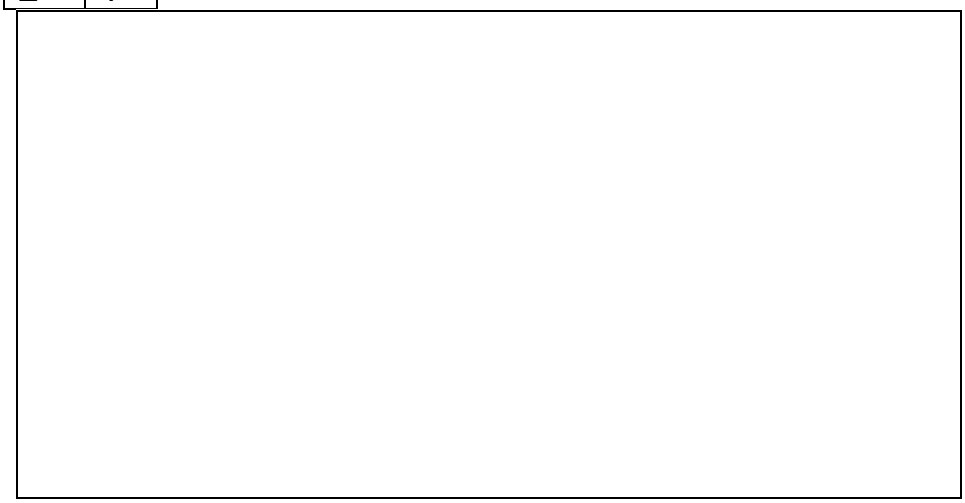
86. exponential graph, shift [ ] one unit down, horizontal asymptote moves down one unit also to the horizontal line y = –1.

x	y
-1	-0.5
0	0
1	1



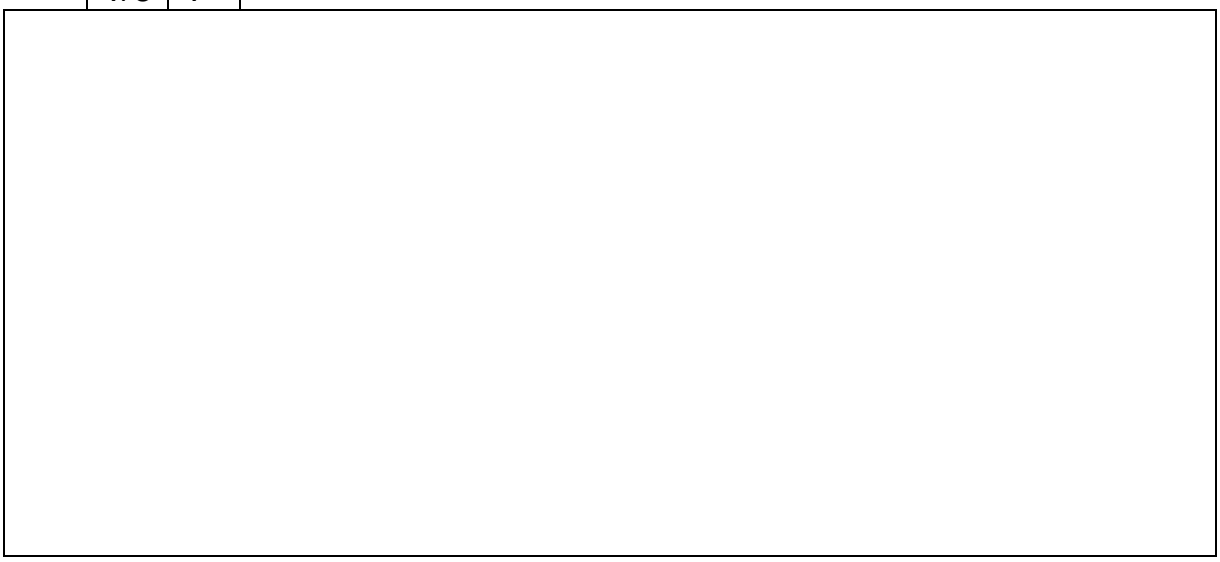
87. logarithmic graph, vertical asymptote is y-axis (x = 0)

x	y
0.5	-1
1	0
2	1



88. logarithmic graph, vertical asymptote is y-axis

x	y
3	-1
1	0
1/3	1



89. graph of square root function, domain is $x > -3$.

x	y
-3	0
1	2
6	3